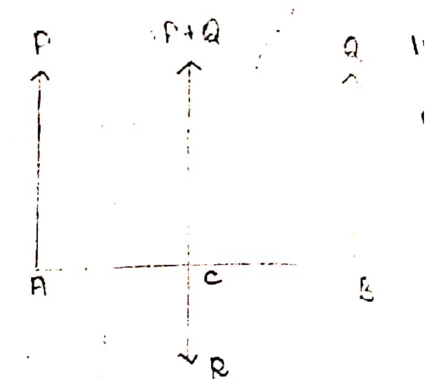
Conditions of equilibrium of three Coplanar parallel forces



Let P, Q, R be three forces parallel in one plane and be in equilibrium. Draw a line to meet the lines of action of these forces at A, B and C respectively

Resultant of P and Q = P+Q

This P+Q is parallel to P or Q. For, equilibrium R must be equal and opposite to (P+Q).

R = P+Q and the line of action of P+Q must passes through C.

$$\therefore P \cdot AC = Q \cdot CB \Rightarrow \frac{P}{CB} = \frac{Q}{AC}.$$

and resultant $= \dfrac{P+Q}{CB+AC} = \dfrac{P+Q}{AB} = \dfrac{R}{AB}$.

$$\therefore \frac{P}{CB} = \frac{Q}{AC} = \frac{R}{AB}.$$

Thus, if 3 parallel forces are in equilibrium, each is proportional to the distance between the other two.

Problems:

1. Two men, one stronger than the other, have to remove a block of stone weighting 300 kgs with a light pole whose length is 6 metre. The weaker man cannot carry more than 100 kgs. Where must the stone be fastened to the pole, so as just to allow him his full share of weight?

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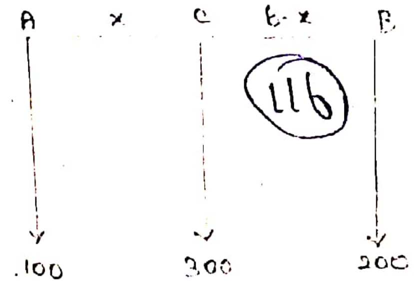
Soln:



Let A be the weakerman bearing 100 kgs. his full share of the weight of the same. and B be the stronger man bearing 200 kgs.

Let C be the point on AB where the stone is fastened to the pole. Such that AC = x.

Then the weight of the same acting at C is the resultant of the parallel forces 100 and 200 at A and B respectively.

$$100\, AC = 200\, BC$$

$$100x = 200(6-x)$$

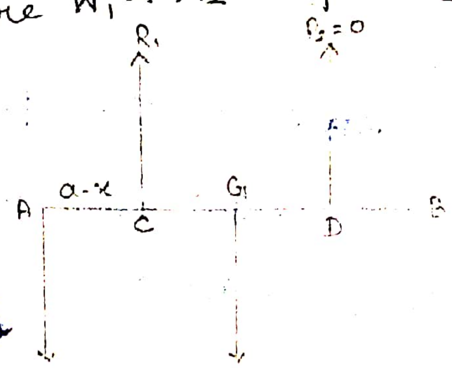
$$= 1200 - 200x$$

$$300x = 1200$$

$$\boxed{x = 4}$$

Hence the stone must be fastened to the pole at the point distance 4 metres from the weakerman.

2. A uniform plank AB of length 2a and weight W is supported horizontaly on two horizontal pegs C & D at a distance 'd' apart. The greatest weight that can be placed at the two ends in succession without upsetting the plank are W_1 & W_2 respectively. S.T $\frac{W_1}{W+W_1} + \frac{W_2}{W+W_2} = d/a$.



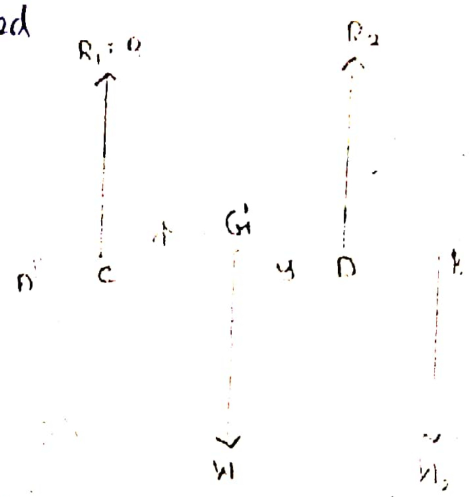
Soln:

Let G be the mass Centre

AG = GB = a and CG = x, GD = y, CD = d

Let R_1, R_2 be the reactions at C, D.

When the greatest weight W_1 is placed at A, R_2 is just zero. Now taking moments about C,



$$(a-x)W_1 - xW = 0$$

$$\Rightarrow \quad x = \frac{aW_1}{W+W_1}$$

Similarly, in the other case, taking moments about D,

$$yW - (a-y)W_2 = 0$$

$$\Rightarrow \quad y = \frac{aW_2}{W+W_2}$$

But $x+y=d$.

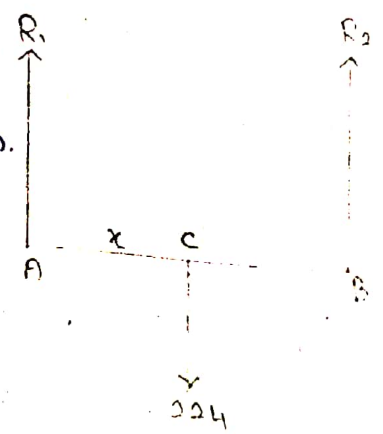
$$\frac{aW_1}{W+W_1} + \frac{aW_2}{W+W_2} = d$$

$$\frac{W_1}{W+W_1} + \frac{W_2}{W+W_2} = d/a.$$

3- Two men carry a load of 224 kg wt. which hangs from a light pole of length 8m each end of which rests on a shoulder of one of the men. The point from which the load is hung is 2m. nearer to one man than the other. What is the pressure on each shoulder?

Soln:

AB is the light pole of length 8cm.



C is the point from which the load of 224 kgs is hung.

Let $AC = x$

Then $BC = 8 - x$

given : $(8-x) - x = 2 \Rightarrow 8 - 2x = 2$

$\Rightarrow \quad -2x = -6$

$\Rightarrow \quad \boxed{x = 3}$

ie) $AC = 3$ and $BC = 5$

Let the pressures at A and B be R_1 and R_2 Kg wt respectively.

Since the pole is in equilibrium, the algebraic sum of the moments of the 3 forces R_1, R_2 and 224 kg wt about any point must be $= 0$.

Taking moments about B,

$224\, CB - R_1 \cdot AB = 0$ (as the moments of R_2 about $B = 0$)

ie) $224 \times 5 - R_1 \times 8 = 0$

$$R_1 = \frac{224 \times 5}{8} = 140.$$

Taking moments about A,

$R_2 AB - 224\, AC = 0$

$8 R_2 - 224 \times 3 = 0$

$R_2 = 84$

Note:

For equilibrium the weight of 224 kgs must be equal and opposite to the resultant of R_1 & R_2.

$R_1 + R_2 = 224$

Hence from this relation, we may find R_2, after finding R_1.

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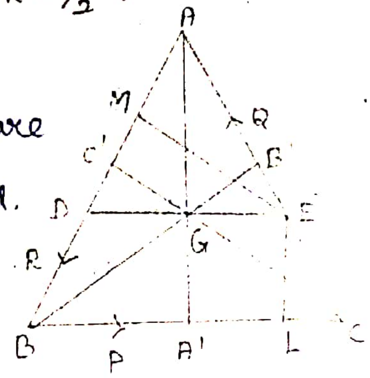
4. Forces P, Q, R act along the sides BC, AC, BA respectively of an equilateral triangle. If their resultant is a force parallel to BC through the centroid of the triangle. P.T $Q = R = \frac{1}{2}P$. (19)



Soln:

$\triangle ABC$ is equilateral. AA', BB', CC' are the medians, altitudes G is the Centroid.

Let DE ∥ BC through G.

given, DGE is the line of action of the resultant. As the resultant passes through G, its moments about G = 0.

∴ Sum of the moments of P, Q, R about G = 0.

ie) $P \cdot GA' - Q \cdot GB' - R\,GC' = 0$

ie) $P - Q - R = 0 \rightarrow (1)$ $\quad [GA' = GB' = GC']$

Since the resultant passes through E also, sum of the moments of P, Q, R about E = 0.

Draw EL ⊥r BC and EM ⊥ AB.

$\therefore P \cdot EL - R \cdot EM = 0 \rightarrow (2)$

From the similar triangles ELC & AA'C,

$\frac{EL}{AA'} = \frac{EC}{AC} = \frac{1}{3}$ $\quad [\because DGE \parallel BC$ and $\frac{AD}{DB} = \frac{AE}{EC} = \frac{AG}{GA'} = \frac{2}{1}$

$\therefore EC = \frac{1}{3} AA' \rightarrow (3)$

From the similar triangles AME and ACC',

$\frac{EM}{CC'} = \frac{AE}{AC} = 2/3$

$\therefore EM = \frac{2}{3} CC' \rightarrow (4)$

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Sub ③ & ④ in ②, we have

$$P \cdot \frac{1}{3} AA' - R \cdot \frac{2}{3} CC' = 0$$

(i.e) $P = 2R$ $[\because AA' = CC']$

$R = P/2 \rightarrow$ ⑤

Putting $R = P/2$ in ①, we have

$$P - Q - \frac{P}{2} = 0$$

$$P/2 - Q = 0$$

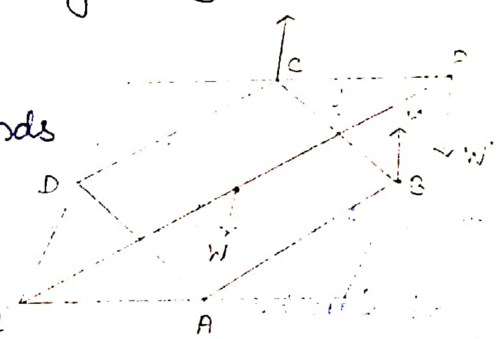
$$P/2 = Q$$

⑤ & ⑥ $\Rightarrow$ $\boxed{R = \frac{P}{2} = Q}$

5: A square table stands on 4 legs placed at the middle points of its sides. find the greatest weight which can be put at one corner of the table without upsetting it, the total weight of the table & legs being W.

Soln:

Let A, B, C, D be the upper ends of the legs.



Let O be the CG of the table.

Let the diagonal QP of the table meet BC at M. Then MB = MC, MO = MP. Let W' be the weight to be put at P. At the moment of tilling the reactions on the legs at A, D are zero.

Taking the moments of the other forces about M, the sum of the moments the equal reactions through B, C = 0 and

$$MO \cdot W - PM \cdot W' = 0$$

$$\boxed{W' = W}$$

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Normal reaction and friction:

We shall now consider the reaction acting on one of these bodies. When this forces is resolved into two components, one along the direction normal to the surface of the body at the point of contact and the other along the direction opposite to the tangential direction in which the body has a tendency to move, the components are called the normal reaction and the friction respectively. It is important to note the difference between reaction and the normal reaction. When the bodies are smooth, which, of course, is an ideal case, the friction itself vanishes and the reaction is the normal reaction itself.

Limiting friction:

It has been observed through experiments that there is a limit to the amount of friction that can be called into play. This maximum limit is called the limiting friction. When one body is just on the point of sliding on another body, the equilibrium is said to be limiting equilibrium and the friction then exerted is the limiting friction. It has also been observed that the limiting friction bears a constant ratio to the normal reaction. This constant is called the coefficient

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of friction. If the limiting friction is F and the normal reaction is R and if the coefficient of friction is μ, then

$$\frac{F}{R} = \mu \quad \text{or} \quad F = \mu R.$$

Angle of friction and Cone of friction:

When the friction is the limiting friction, the angle between the reaction & the normal to the Surface is called the angle of friction. The right Circular Cone with its Vertex at the point of Contact, with its axis along the normal to the surface and with its Semi-Vertical angle equal to the angle of friction is called the Cone of friction. If the angle of friction is λ, then

$$\tan\lambda = \frac{\text{limiting friction}}{\text{Normal reaction}} = \mu \text{ or } \lambda = \tan^{-1}\mu$$

Law of friction:

It is observed from experiments that the nature of friction is governed by the following laws of friction.

Law 1:

The friction acts opposite to the direction in which the body moves or has a tendency to move (relative to the other body in Contact).

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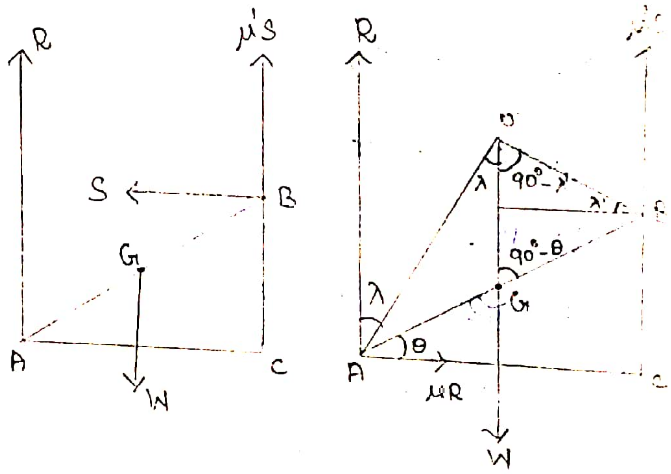
$$= \frac{\sin(\alpha+\lambda-\alpha+\lambda)}{2\cos(\alpha+\lambda)\cos(\alpha-\lambda)}$$

$$\theta = \tan^{-1}\left[\frac{\sin 2\lambda}{\cos 2\alpha + \cos 2\lambda}\right]$$

2. (i) A ladder is in equilibrium with one end resting on the ground and the other end against a vertical wall, if the ground and the wall are both rough, the coefficient of friction being μ and μ'. And if the ladder is on the point of slipping S.T the inclination of the ladder to the horizontal given by $\tan\theta = \frac{1-\mu\mu'}{2\mu}$.

(ii) When $\mu = \mu'$, S.T $\theta = 90^\circ - 2\lambda$, where λ is the angle of friction.

Soln:



(i) OG is vertical.

In ΔABC,

$\angle OGB = 90^\circ - \theta$

$\angle AOG = \lambda$

$\angle BOG = 90^\circ - \lambda'$

$AG : GB = 1:1$

$(1+1)\cot(90^\circ - \theta) = \cot\lambda - \cot(90^\circ - \lambda')$

$2\tan\theta = \cot\lambda - \tan\lambda'$

$= \frac{1}{\mu} - \mu'$

$\tan\theta = \frac{1-\mu\mu'}{2\mu}$

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(ii) AD- plane.

BD- Vertical Wall

AB- ladder

In ΔAOB

$\underline{|AOG} = \lambda$, $\underline{|BOG} = 90^\circ - \lambda$

$\underline{|OGB} = 90^\circ - \theta$, $AG : GB = 1:1$

$$(1+1)\cot(90^\circ - \theta) = 1\cdot\cot\lambda - 1\cdot\cot(90^\circ - \lambda)$$

$$2\tan\theta = \cot\lambda - \tan\lambda$$

$$2\tan\theta = \frac{1}{\mu} - \mu$$

$$\tan\theta = \frac{1-\mu^2}{\mu} \quad \text{Since } \mu = \mu'$$

$$\tan\theta = \frac{1-\mu^2}{\mu} = \frac{1-\tan^2\lambda}{2\tan\lambda}$$

$$\tan\theta = \cot 2\lambda = \tan(90^\circ - 2\lambda)$$

$$\theta = \pi/2 - 2\lambda$$

$$\boxed{\theta = 90^\circ - 2\lambda}$$

3. A ladder AB with A resting on the ground and B against a Vertical wall, the Co-efficient of friction of the ground and the wall being μ and μ' respectively The Centre of gravity G of the ladder divides AB in the ratio $1:n$. If the ladder is on the point of slippings at both ends, S.T its inclination to the ground is given by $\tan\theta = \frac{1-n\mu\mu'}{(n+1)\mu}$.

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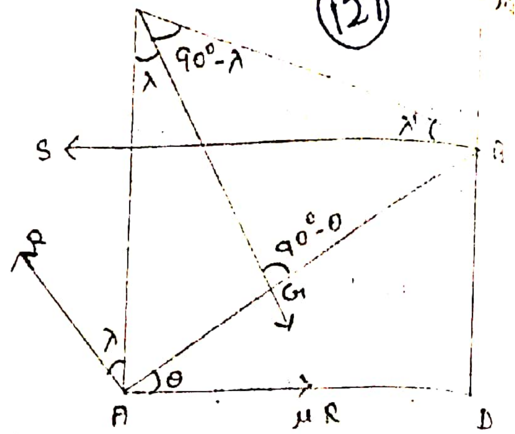
Soln: :

AD - horizontal plane

BD - Vertical wall

AB - ladder



AG : GB = 1 : n

$(1+n)\cot(90°-\theta) = 1\cdot\cot\lambda - n\cot(90°-\lambda')$

$$(1+n)\tan\theta = \cot\lambda - n\tan\lambda'$$

$$= \frac{1}{\tan\lambda} - n\tan\lambda'$$

$$= \frac{1}{\mu} - n\mu'$$

$$\tan\theta = \frac{1-n\mu\mu'}{(n+1)\mu}$$

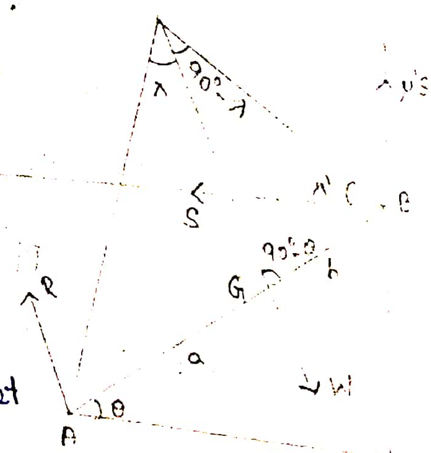
4. A ladder AB rests with A on a rough horizontal ground and B against an equally rough Vertical wall. The C.G of the ladder divides AB in the ratio a:b. If the ladder is on the point of Slipping, S.T the inclination θ of the ladder to the ground is given by $\tan\theta = \frac{a-b\mu^2}{\mu(a+b)}$, where μ is the Co-efficient of friction.

Soln:

AB - ladder

AG : GB = a : b

$\mu = \tan\lambda$, $\mu' = \tan\lambda'$



Forces acting normally at A, B meet at O. And made by A to the Vertices is λ.

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W acts along o. ie) oG is Vertical

θ - angle made by the ladder to the floor

In ΔOAB:

$\underline{\angle AOG} = \lambda$, $\underline{\angle BOG} = 90^\circ - \lambda'$, $\underline{\angle OGB} = 90^\circ - \theta$

$$\therefore (a+b)\cot(90^\circ - \theta) = a\cot\lambda - b\cot(90^\circ - \lambda')$$

$$(a+b)\tan\theta = \frac{a}{\tan\lambda} - b\tan\lambda'$$

$$= \frac{a}{\mu} - b\mu'$$

$$(a+b)\tan\theta = \frac{a - b\mu\mu'}{\mu}$$

$$\tan\theta = \frac{a - b\mu\mu'}{(a+b)\mu} \qquad (\because \mu = \mu')$$

$$\theta = \tan^{-1}\left(\frac{a - b\mu^2}{(a+b)\mu}\right)$$

5. A uniform ladder rests at an angle 45° with the horizontal, with its upper end against a wall and its lower end on the ground. If μ and μ' are the coefficient of friction of the ground and the wall. S.T the least horizontal force which will have the lower extremity towards the wall is $\frac{1}{2}W\left(\frac{1+2\mu+\mu\mu'}{1-\mu'}\right)$

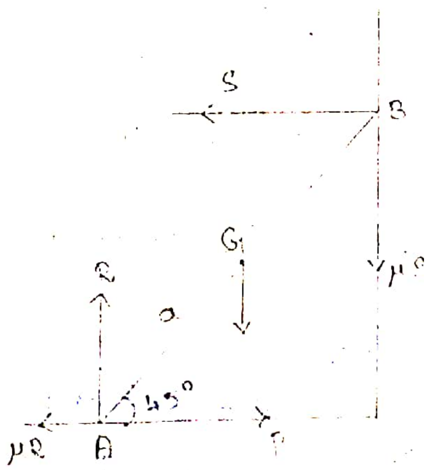
Soln:

AB - ladder

AB = 2a (say)

G - C.G

AG = GB = a



Let the n) reactions A, B be R & S

P - horizontal forces of friction

Resolving forces horizontally & Vertically,

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$$P - \mu R - S = 0 \Rightarrow P = \mu R + S \rightarrow ①$$

$$R - W - \mu' S = 0 \Rightarrow R = W + \mu' S \rightarrow ②$$

Taking moments about A,

$$-W \cdot AL - \mu' S \, AO + S \cdot OB = 0$$

$$\Rightarrow S \cdot 2a \sin 45^\circ = Wa \cos 45^\circ + \mu' S \cdot 2a \cos 45^\circ$$

$$2S = W + 2\mu' S$$

$$S = \frac{W}{2(1-\mu')}$$

Sub in ①

$$\Rightarrow P = \mu R + \frac{W}{2(1-\mu')}$$

$$= \mu \left[W + \frac{\mu' W}{2(1-\mu')} \right] + \frac{W}{2(1-\mu')}$$

$$= \frac{W}{2(1-\mu')} \left[2(1-\mu')\mu + \mu\mu' + 1 \right]$$

$$= \frac{W}{2(1-\mu')} \left[2\mu - 2\mu\mu' + \mu\mu' + 1 \right]$$

$$P = \frac{1}{2} W \left[\frac{1 + 2\mu - \mu\mu'}{1 - \mu'} \right]$$

6. A solid hemisphere rests in equilibrium on a rough ground & against an equally rough vertical wall, the coefficient of friction being μ, S.T if the equilibrium is limiting, the inclination of the base to the horizontal is $\sin^{-1} \frac{8\mu(1+\mu)}{3(1+\mu^2)}$.

Soln:

c - Centre of hemisphere

a - radius

G - C.G of hemisphere

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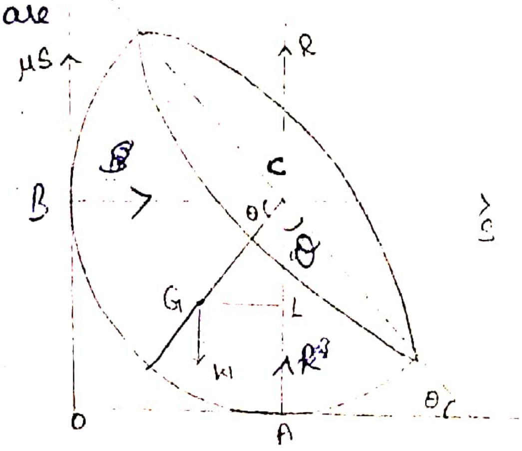
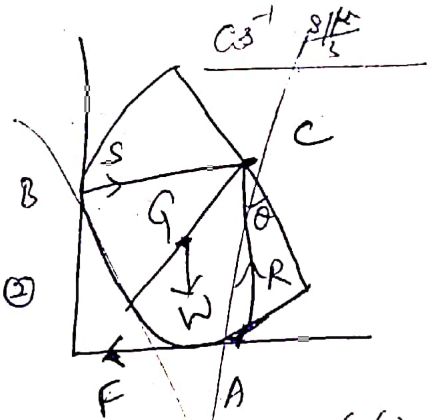
$GC = \frac{3}{8}a.$

The forces acting on the hemisphere are



1. Angle of friction μR against wall at A.
2. Normal reaction R vertically to the floor at A.
3. Angle of friction μS vertically upwards at B.
4. Normal reaction S vertically to the floor at B.
5. Weight W through G vertically downwards.

Resolving forces horizontally



$$\mu R - S = 0 \Rightarrow S = \mu R \rightarrow (1)$$

Resolving forces vertically

$$\mu S + R - W = 0 \Rightarrow R = W - \mu S \rightarrow (2)$$

(2) in (1) $\Rightarrow S = \mu(W - \mu S)$

$$S = \mu W - \mu^2 S$$

$$S(1+\mu^2) = \mu W$$

$$S = \frac{W\mu}{1+\mu^2} \rightarrow (3)$$

Let, in the equilibrium position, the points of contact with the plane and the wall be A, B. The forces on the body are
(i) Friction F at A
(ii) Reaction R " "
(iii) " S " B.
(iv) Weight W at G.

Taking moments at A,

$$S \cdot CA + \mu \cdot S \cdot AO - W \cdot GL = 0$$

Let θ - angle between floor and bottom of hemisphere

$\sin\theta = \frac{GL}{GC} = \frac{GL}{(3a/8)}$

$$\therefore GL = \frac{3}{8} a \sin\theta$$

$AC = a$

$AO = CB = a$

$$\therefore S \cdot a + \mu \cdot S \cdot a = W \cdot \frac{3}{8} a \sin\theta$$

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$\div$ by a;

(13)

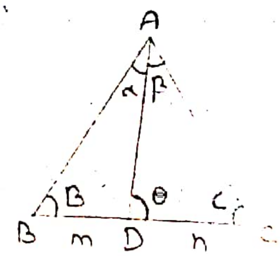
$$\Rightarrow S + \mu S = \frac{3}{8} W \sin\theta$$

$$\frac{3}{8} W \sin\theta = S(1+\mu)$$

$$= \frac{(1+\mu)\cdot W\mu}{1+\mu^2}$$

$$\sin\theta = \frac{8}{3} \frac{\mu(1+\mu)}{1+\mu^2}$$

$$\theta = \sin^{-1} \frac{8\mu(1+\mu)}{3(1+\mu^2)}$$



+ **Cotangent formula:**

ABC is triangle

D is a point on BC dividing BC in ratio m:n

$\angle ADC = \theta$, $\angle BAD = \alpha$, $\angle CAD = \beta$.

$$\therefore (m+n)\cot\theta = m\cot\alpha - n\cot\beta$$

$$= n\cot B - m\cot C$$

7. A uniform ladder AB rests in limiting equilibrium with the end A on a rough floor, the coefficient of friction being μ and with the other end B against a smooth vertical wall. S.T if θ is the inclination of the ladder to the vertical, then

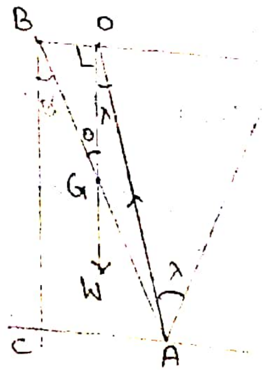
(i) $\tan\theta = 2\mu$

(ii) If $\theta = 30°$, find μ.

Soln:

i) AB - uniform ladder

G - Centre of gravity



AG : GB = 1:1

λ - angle of friction.

Let the weight W and the reaction at B meet at O.

Then the resultant reaction of the normal reaction & force of friction at A passes through O. (132)

The reaction is inclined to the normal to the floor.

ie) To the Vertical at an angle λ.

In ΔAGB, $AG : GB = 1 : 1$

$\angle OGB = \theta$; $\angle GOB = 90^\circ$, $\angle GOA = \lambda$.

By Cotangent formula

$m = 1$, $n = 1$

$$(1+1)\cot\theta = \cot\lambda - \cot 90^\circ$$

$$2\cot\theta = \cot\lambda$$

$$\tan\theta = 2\tan\lambda$$

$$\tan\theta = 2\mu$$

(ii) $\theta = 30^\circ$

$$\tan 30^\circ = 2\mu$$

$$\mu = \frac{1}{2}\tan 30^\circ$$

$$\mu = \frac{1}{2\sqrt{3}}$$

8. A ladder of length $2a$ is in Constant with a wall & a horizontal floor, the angle of friction being λ at each Contact. If the Centre of gravity of the ladder is at a distance ka below the midpoint, S.T in the limiting equilibrium, the inclination θ to the Vertical is given by $\cot\theta = \cot 2\lambda - k\operatorname{cosec} 2\lambda$.

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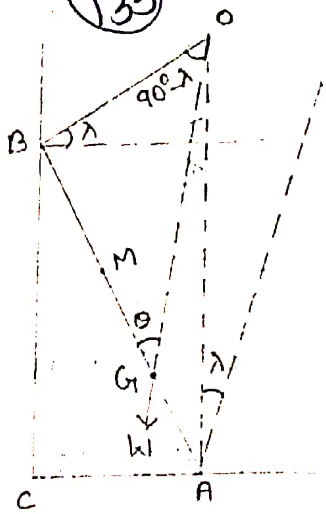
Soln:



AB- uniform ladder

G- Centre of gravity of ladder

AB= 2a

The forces acting on the ladder are:

* The reactions of the floor
* The wall inclined to the normals to them at an angle λ.
* The weight. W

If they meet at o, then both AO, and BO are inclin to the Vertical & horizontal at the same angle λ.

$\therefore$ In Δ^{le} ABO, $\underline{BGO} = \theta$, $\underline{AOG} = \lambda$, $\underline{BOG} = 90^\circ - \lambda$

$BG = a + ka$, $AG = a - ka$ $(\because GM = ka)$

$\therefore BG : AG = a + ka : a - ka$

$= 1 + k : 1 - k$

By Cotangent formula,

$$[(1+k)+(1-k)] \cot\theta = (1-k)\cot\lambda - (1+k)\cot(90^\circ - \lambda)$$

$$2\cot\theta = (1-k)\cot\lambda - (1+k)\tan\lambda$$

$$2\cot\theta = \cot\lambda - k\cot\lambda - \tan\lambda - k\tan\lambda$$

$$2\cot\theta = (\cot\lambda - \tan\lambda) - k(\cot\lambda + \tan\lambda)$$

W.k.t

$$\cot\lambda - \tan\lambda = 2\cot 2\lambda$$

$$\cot\lambda + \tan\lambda = 2\,\mathrm{cosec}\, 2\lambda$$

$$\Rightarrow 2\cot\theta = \frac{1}{\tan\lambda} - \tan\lambda - k\left(\frac{\cos\lambda}{\sin\lambda} + \frac{\sin\lambda}{\cos\lambda}\right)$$

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$$= \frac{1-\tan^2\lambda}{\tan\lambda} - k\left(\frac{1}{\sin\lambda\cos\lambda}\right)$$

$$= \frac{2}{\tan 2\lambda} - k\left(\frac{2}{\sin 2\lambda}\right)$$

$$2\cot\theta = 2\cot 2\lambda - 2k\,\mathrm{cosec}\,2\lambda$$

$$\boxed{\cot\theta = \cot 2\lambda - k\,\mathrm{cosec}\,2\lambda}$$

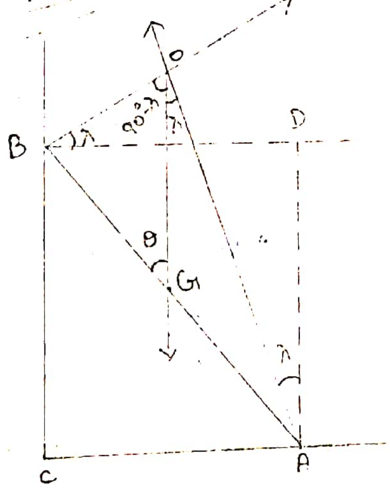
9. Find the inclination θ to the vertical of a uniform ladder AB of length $2a$ and weight W which is in limiting equilibrium having contact with a rough horizontal floor and a rough vertical wall, the coefficient of friction being μ. S.T the greatest inclination of the ladder to the vertical is 2λ.

Soln:

AB - ladder

AC - horizontal plane

BC - vertical wall



Forces on the rod are:

* normal reaction at A.
* reactions of the wall.
* weight.

Let them meet at O.

Then AO is inclined to the normal AD to the floor at an angle λ.

BO is inclined to the normal BD to the wall at an angle λ.

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In Δ^{le} OAB, BG : GA = 1 : 1

$\angle OGB = \theta$, $\angle BOG = 90^\circ - \lambda$, $\angle AOG = \lambda$

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By Cotangent formula,

$$(1+1)\cot\theta = \cot\lambda - \cot(90^\circ - \lambda)$$

$$= \cot\lambda - \tan\lambda$$

$(\mu = \tan\lambda \Rightarrow \frac{1}{\mu} = \cot\lambda)$

$$2\cot\theta = \frac{1}{\mu} - \mu$$

$$2\cot\theta = \frac{1-\mu^2}{\mu}$$

$$\frac{\tan\theta}{2} = \frac{\mu}{1-\mu^2}$$

$$\boxed{\tan\theta = \frac{2\mu}{1-\mu^2}}$$

(ii) Since $\mu = \tan\theta$

$$\tan\theta = \frac{2\tan\lambda}{1-\tan^2\lambda}$$

$$\tan\theta = \tan 2\lambda$$

$$\boxed{\theta = 2\lambda}$$

10. A uniform ladder of length l rest on a rough horizontal ground with its upper end projecting slightly over a smooth horizontal rod at a height h above the ground. If the ladder is about to slip, S.T the Coefficient of friction μ is equal to $\frac{h\sqrt{l^2-h^2}}{l^2+h^2}$.

Soln:

AB - ladder

AC - horizontal ground.

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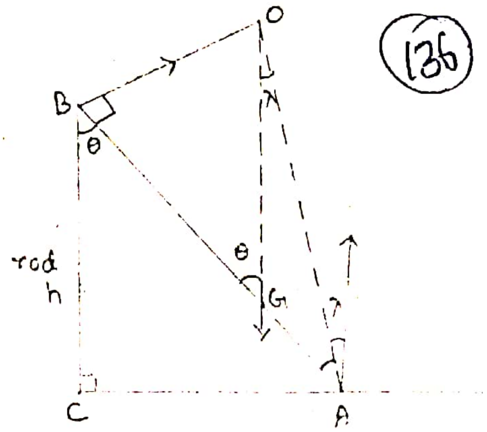
BC - Smooth rod

AB = l

BC = h



Let the 3 forces acting on the ladder meet at O.

$AG = BG = l/2$

$AG : GB = 1:1$

In ΔABO, $\angle OGB = \theta$, $\angle OBA = 90^\circ$

$\angle BOG = 90^\circ - \theta$, $\angle AOG = \lambda$

In ΔABC, $AB = l$, $BC = h$

$\therefore AC = ?$, $AC^2 = AB^2 - BC^2$

$$AC = \sqrt{l^2 - h^2}$$

Now,

$$2\cot\theta = \cot\lambda - \cot(90^\circ - \theta)$$

$$= \cot\lambda - \tan\theta$$

$$\cot\lambda = 2\cot\theta + \tan\theta$$

$$= 2 \cdot \frac{h}{\sqrt{l^2 - h^2}} + \frac{\sqrt{l^2 - h^2}}{h}$$

$$= \frac{2h^2 + l^2 - h^2}{h\sqrt{l^2 - h^2}}$$

$$= \frac{l^2 + h^2}{h\sqrt{l^2 - h^2}}$$

$$\frac{1}{\tan\lambda} = \frac{l^2 + h^2}{h\sqrt{l^2 - h^2}}$$

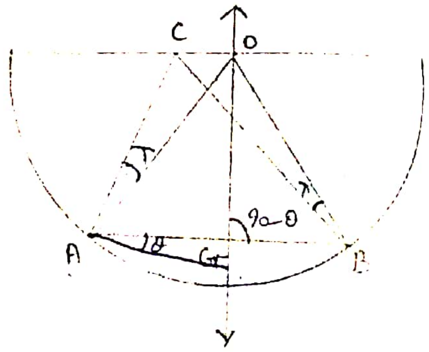
$$\tan\lambda = \frac{h\sqrt{l^2 - h^2}}{l^2 + h^2} = \mu$$

11. A uniform rod AB rests within a fixed hemispherical bowl whose radius is equal to length of the rod. If μ is the Coefficient of friction, S.T on limiting equilibrium, the inclination θ of the rod to the horizontal is given by $\tan\theta = \frac{4\mu}{3-\mu^2}$.

Soln:

AB - uniform rod

AC, BC - radii of the hemispherical bowl



$\therefore$ radii AC, BC and the rod AB form an equilateral triangle.

given,

radius of bowl = length of rod.

If the reactions of the bowl A, B and weight of the rod meet at O, then

AO, BO are inclined to AC, BC at the same angle of friction λ.

In ΔAOB, $AG : GB = 1:1$

$\underline{\angle OGB} = 90° - \theta$, $\underline{\angle GAO} = 60° - \lambda$, $\underline{\angle GBO} = 60° + \lambda$

$\therefore$ By Cotangent formula,

$$(1+1)\cot(90°-\theta) = \cot(60°-\lambda) - \cot(60°+\lambda)$$

$$2\tan\theta = \frac{1+\tan 60°\tan\lambda}{\tan 60° - \tan\lambda} - \frac{1-\tan 60°\tan\lambda}{\tan 60° + \tan\lambda}$$

$$= \frac{1+\sqrt{3}\mu}{\sqrt{3}-\mu} - \frac{1-\sqrt{3}\mu}{\sqrt{3}+\mu}$$

$$= \frac{(1+\sqrt{3}\mu)(\sqrt{3}+\mu) - (1-\sqrt{3}\mu)(\sqrt{3}-\mu)}{3-\mu^2}$$

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$$= \frac{\sqrt{3}+\mu+3\mu+\sqrt{3}\mu^2-\sqrt{3}+\mu+3\mu-\sqrt{3}\mu^2}{3-\mu^2}$$

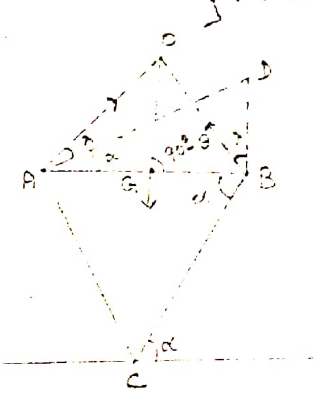
$\therefore$

$$2\tan\theta = \frac{2\mu+6\mu}{3-\mu^2} = \frac{8\mu}{3-\mu^2}$$

$$\boxed{\tan\theta = \frac{4\mu}{3-\mu^2}}$$

12. A rod is in limiting equilibrium resting horizontally with its ends on two inclined planes which are at right angles & one of which makes an angle α ($<45°$) with the horizontal. If the coefficient of friction is the same for both the ends, S.T $\mu = \frac{1-\tan\alpha}{1+\tan\alpha}$.

Soln:



The forces acting on the rod are

i) The reactions of the planes at A & B

ii) Weight W

Let the reactions meet at O.

Then OG is Vertical. Also OA = OB.

Hence $\angle OAG = \angle OBG$. A has a tendency to slide downwards, and upwards because $\alpha < 45°$.

AO, & OB are inclined to AC, BC at the same angle of friction λ.

In ΔAOB, AG : GB = 1 : 1

$\angle GAO = \alpha+\lambda$, $\angle OBG = 90° - (\alpha+\lambda)$

$\alpha+\lambda = 90° - (\alpha+\lambda) \Rightarrow \alpha+\lambda = 45°$

$$\Rightarrow \tan(\alpha+\lambda) = \tan 45^\circ = 1$$

$$\frac{\tan\alpha + \tan\lambda}{1-\tan\alpha\tan\lambda} = 1$$

$$\tan\alpha + \mu = 1 - \mu\tan\alpha$$

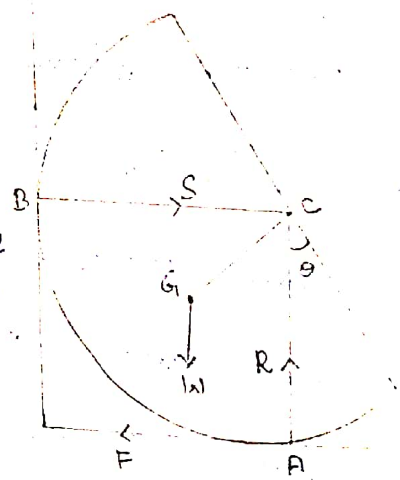
$$\mu(1+\tan\alpha) = 1-\tan\alpha$$

$$\mu = \frac{1-\tan\alpha}{1+\tan\alpha}$$

13. A solid hemisphere rests on a rough horizontal plane and against a smooth Vertical wall. S.T if the Coefficient of friction μ is greater than $3/8$, then the hemispheres Can rest in any position & if it is less, the least angle that the base of the hemisphere Can make with the vertical is $\cos^{-1}\frac{8\mu}{3}$.

Soln:

In equilibrium position,



let A → Pot of Contact with the plane

& B → Pot of Contact with the wall

The forces on the body are

(i) Friction F at A

(ii) Reaction R at A

(iii) Reaction S at B

(iv) Weight W at G.

Then, from the horizontal and Vertical Components.

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$S - F = 0$, $R - W = 0$

$\Rightarrow S = F$, $R = W$

Now $CG = \frac{3a}{8}$, where $a \rightarrow$ radius of the sphere

The distance of G from a Vertical through A is

$CG \cos\theta = \frac{3a}{8} \cos\theta$

where θ is the angle made by the base with the Vertical. Thus taking moments about A,

$\frac{3a}{8} \cos\theta \, W - a \cdot S = 0$

$\frac{3a}{8} \cos\theta \cdot R = aF$

$\frac{F}{R} = \frac{3}{8} \cos\theta.$

(i) So the equilibrium is possible for any inclination so long the Coefficient of friction is large such that

$\mu > \frac{3}{8} \cos\theta \quad [\because \mu = F/R]$

$\Rightarrow \frac{8\mu}{3} > \cos\theta \rightarrow (1)$

But the maximum Value for $\cos\theta$ is 1.

Thus $1 \not> \frac{8\mu}{3} > 1$

$\mu > 3/8$

(ii) If $\mu < 3/8$ and if the equilibrium is limiting, then

$F = \mu R$ & (1) gives

$\mu = 3/8 \cos\theta$

$\Rightarrow \cos\theta = \frac{8\mu}{3}$

$\theta = \cos^{-1}\left(\frac{8\mu}{3}\right)$

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Which is the least angle between for a ~~still~~ still

Smaller angle. the friction is larger which Cannot be

attained.

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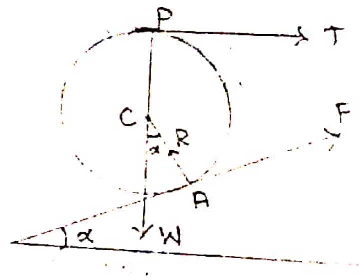
14. A sphere of weight W resting on a rough incline plane of inclination α is kept in equilibrium by a horizontal string attached to the highest point of the sphere. S.T the angle of friction is greater than $\alpha/2$ and that the tension of the string is $W\tan\alpha/2$.

Soln:

A - The point of Contact

C - the Centre

P - The top most point



Forces on the sphere are

(i) friction F at A up the plane.

(ii) Reaction R at A normal to the plane

(iii) Tension T horizontally at P

(iv) Weight W vertically downward.

Taking moments about c,

$$aF - aT = 0$$

$$F = T \rightarrow ①$$

Resolving the forces horizontally & Vertically

$$T + F\cos\alpha = R\sin\alpha \rightarrow ②$$

$$R\cos\alpha + F\sin\alpha = W \rightarrow ③$$

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① & ②

$$\frac{F}{R} = \frac{\sin\alpha}{1+\cos\alpha}$$

$$= \frac{2\sin\alpha/2\,\cos\alpha/2}{2\cos^2\alpha/2}$$

$$= \tan\alpha/2$$

μ should be large enough so that $\frac{F}{R} < \mu$

ie) $\tan\alpha/2 < \tan\lambda$

$\Rightarrow \alpha/2 < \lambda$

Since $F=T$ and solving for T, we get

$$T = \frac{\sin\alpha}{1+\cos\alpha} W$$

$$T = W\tan\alpha/2.$$

Friction: If 2 bodies are in contact
like property of the 2 bodies by
one another, which a force is exerted b/w them
means of pt of contact to prevent
at these from sliding on the other is
one body friction, called the force of friction
called exerted is
the force

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